

ANSWERS TO CHAPTER QUESTIONS

Chapter 2 The Time Value of Money

- 1) Compounding is interest paid on principal and interest accumulated. It is important because normal compounding over many years can result in a more accurate and greater accumulated sum at the end of the period than what may have been anticipated. On the other hand, returns on accumulated sums can be appreciably higher under compounding than calculated through simple return methods.
- 2) It is important to assess the value of a sum of money at different points in time. Among other things, it leads to incorporation of the required return on monies invested in forming decisions. These decisions may be too complex to determine through simple guesstimates and could lead to wrong conclusions.
- 3) The present value is the value today of sums to be paid in the future. The value is established by taking future cash flows and discounting them back to the present at an appropriate rate of return. The future value is the accumulated sum at the end of the period. It is calculated by taking cash flows prior to that time frame and compounding them by the appropriate rate of return.
- 4) The rate of return that could be received on marketable investments having the same level of risk.
- 5) When a discount rate is raised, the present value of a future sum is reduced. Alternative investments are now providing a higher return which makes the future sum to be received on the investment being considered less valuable.
- 6) The lump sum today. The reason is the lump sum today has more compounding periods. Assuming a similar market established rate of return for both, a sum invested

in the future will have a lower present value than one that exists today and a lump sum invested today will have a greater future value as well.

7) A regular annuity is a series of payments made or received at the end of the period. An annuity due indicates payments made or received at the beginning of the period. Annuity dues have higher values because they have one full period more of compounding. An example of an annuity due is annual payments made on January 1 each year as contributions toward retirement. Annual payments received on December 31 are an example of a regular annuity.

8) The rate of return is the sum you receive expressed as compensation to you for making an investment. An inflation-adjusted return adjusts for a rise in the cost of living. Making that adjustment allows returns to be expressed in purchasing power terms. Doing so is particularly important in personal financial planning which uses investments to fund future expenditures with these future costs often rising with inflation.

9) When payments are due at the end of the period they are called a regular annuity. When payments are due at the beginning of the period they are called an annuity due.

10) The Rule of 72 gives a quick estimate on when your investment return will double based on the investment return percentage.

11) Future value is the value that a set amount of money will be worth using today's dollars and discounted by the rate of inflation.

a)
$$\text{Future value} = \text{Cash Flow} \times (1 + \text{interest rate})^{\text{number of periods}}$$

12) The consequence of not accounting for inflation means not accounting for the decrease in the purchasing power of the dollar. That same dollar that could have

bought you a candy bar today may only be able to purchase half a candy bar 10 years from now.

13) The internal rate of return takes into account the time valuation of money, and cash inflows and outflows. The IRR is often used to determine the profitability of a capital expenditure.

ANSWERS TO CHAPTER PROBLEMS

Chapter 2 Time Value of Money

- 1) What is the present value of a \$20,000 sum to be given 6 years from now if the discount rate is 8 percent?

Excel Solution

| | A | B | C | D |
|----|------------------|------------|-----------------|---|
| 6 | Inputs | | | |
| 7 | Future Cash Flow | \$20,000 | | |
| 8 | Discount Rate | 8% | | |
| 9 | Number of Years | 6 | | |
| 10 | | | | |
| 11 | Solution | | | |
| 12 | Present Value | (\$12,603) | =PV(B8,B9,0,B7) | |

Calculator Solution

| | | | | | |
|---------------|----------|------------|-----------|------------|-----------|
| <i>Inputs</i> | 6 | 8 | | | 20,000 |
| | N | I/Y | PV | PMT | FV |

Solution **-12,603**

- 2) What is the future value of an investment of \$18,000 that will earn interest at 6 percent and fall due in 7 years?

Excel Solution

| | A | B | C | D |
|----|-------------------|----------|------------------|---|
| 6 | Inputs | | | |
| 7 | Present Cash Flow | \$18,000 | | |
| 8 | Interest Rate | 6% | | |
| 9 | Number of Years | 7 | | |
| 10 | | | | |
| 11 | Solution | | | |
| 12 | Future Value | \$27,065 | =FV(B8,B9,0,-B7) | |

Calculator Solution

Inputs 7 6 -18,000

N
I/Y
PV
PMT
FV

Solution **27,065**

3) Jason was promised \$48,000 in 10 years if he would deposit \$14,000 today. What would his compounded annual return be?

Excel Solution

| | A | B | C | D |
|----|-------------------|----------|--------------------|---|
| 6 | Inputs | | | |
| 7 | Present Cash Flow | \$14,000 | | |
| 8 | Future Cash Flow | \$48,000 | | |
| 9 | Number of Years | 10 | | |
| 10 | | | | |
| 11 | Solution | | | |
| 12 | Annual Return | 13% | =RATE(B9,0,-B7,B8) | |

Calculator Solution

Inputs 10 -14,000 48,000

N
I/Y
PV
PMT
FV

Solution **13**

4) How many years would it take for a dollar to triple in value if it earns a 6 percent rate of return?

Excel Solution

| | A | B | C | D |
|----|-----------------|-----|---|---|
| 6 | Inputs | | | |
| 7 | Present Value | \$1 | | |
| 8 | Future Value | \$3 | | |
| 9 | Interest Rate | 6% | | |
| 10 | | | | |
| 11 | Solution | | | |
| 12 | Number of Years | 19 | | |

=NPER(B9,0,-B7,B8)

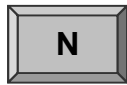
Calculator Solution

Inputs

6

-1

3



Solution 19

5) Marcy placed \$3,000 a year into an investment returning 9 percent a year for her daughter's college education. She started when her daughter was 2. How much did she accumulate by her daughters 18th birthday?

Excel Solution

| | A | B | C | D |
|----|-----------------|----------|---|---|
| 7 | Inputs | | | |
| 8 | Payment | \$3,000 | | |
| 9 | Interest Rate | 9% | | |
| 10 | Number of Years | 16 | | |
| 11 | | | | |
| 12 | Solution | | | |
| 13 | Future Value | \$99,010 | | |

=FV(B9,B10,-B8,0)

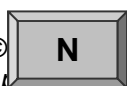
Calculator Solution

Inputs

16

9

-3,000



Solution

99,010






- 6) Todd was asked what he would pay for an investment that offered \$1,500 a year for the next 40 years. He required an 11 percent return to make that investment. What should he bid?

Excel Solution

| | A | B | C | D |
|----|-----------------|------------|------------------|---|
| 7 | Inputs | | | |
| 8 | Payment | \$1,500 | | |
| 9 | Interest Rate | 11% | | |
| 10 | Number of Years | 40 | | |
| 11 | | | | |
| 12 | Solution | | | |
| 13 | Present Value | (\$13,427) | =PV(B9,B10,B8,0) | |

Calculator Solution

Inputs 40 11 1,500

Solution -13,427






- 7) Ann was offered an annuity of \$20,000 a year for the rest of her life. She was 55 at the time and her life expectancy was 84. The investment would cost her \$180,000. What would the return on her investment be?

Excel Solution

| | A | B | C | D |
|----|-----------------|-----------|---------------------|---|
| 7 | Inputs | | | |
| 8 | Payment | \$20,000 | | |
| 9 | Present Value | \$180,000 | | |
| 10 | Number of Years | 29 | | |
| 11 | | | | |
| 12 | Solution | | | |
| 13 | Rate of Return | 10.5% | =RATE(B10,B8,-B9,0) | |

Calculator Solution

Inputs 29 -180,000 20,000

Solution **10.5**

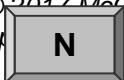
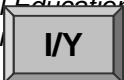



8) How many years would it take for \$2,000 a year in savings earning interest at 6 percent to amount to \$60,000?

Excel Solution

| | A | B | C | D |
|----|-----------------|----------|--------------------|---|
| 6 | Inputs | | | |
| 7 | Payment | \$2,000 | | |
| 8 | Future Value | \$60,000 | | |
| 9 | Interest Rate | 6% | | |
| 10 | | | | |
| 11 | Solution | | | |
| 12 | Number of Years | 18 | =NPER(B9,-B7,0,B8) | |

Calculator Solution

Inputs 6 -2,000 60,000

| | A | B | C | D | E | F |
|----|-----------------------------------|-------------------------|-------------|---|---|---|
| 6 | Inputs | | | | | |
| 7 | Present Value | \$10,000 | | | | |
| 8 | Annual Interest Rate | 12% | | | | |
| 9 | | | | | | |
| 10 | | | | | | |
| 11 | Solution | | | | | |
| 12 | Comparison of Accumulated Amounts | | | | | |
| 13 | | | | | | |
| 14 | Frequency | Periods per Year | FV | | | |
| 15 | Annual | 1 | \$11,200.00 | | | |
| 16 | Monthly | 12 | \$11,268.25 | | | |
| 17 | | | | | | |
| 18 | Difference in Amounts | \$68.25 | | | | |

Calculator Solution

Annual Compounding:

Inputs 1 12 -10,000



Solution

11,200

Monthly Compounding:

Inputs 12 1 -10,000



Solution

11,268.25

Difference in Amounts = 11,268.25 - 11,200 = 68.25

11) What is the difference in future value between savings in which \$3,000 is deposited each year at the beginning of the period and the same amount deposited at the end of the period? Assume an interest rate of 8 percent and that both are due at the end of 19 years.

Excel Solution






| | A | B | C | D | E |
|----|---|-----------|---------------------|---|---|
| 7 | Inputs | | | | |
| 8 | Payment | \$3,000 | | | |
| 9 | Interest Rate | 8% | | | |
| 10 | Number of Years | 19 | | | |
| 11 | | | | | |
| 12 | Solution | | | | |
| 13 | 1) Deposit at the beginning of the period | | | | |
| 14 | Future Value | \$134,286 | =FV(B9,B10,-B8,0,1) | | |
| 15 | | | | | |
| 16 | 2) Deposit at the end of the period | | | | |
| 17 | Future Value | \$124,339 | =FV(B9,B10,-B8,0,0) | | |
| 18 | | | | | |
| 19 | Difference in Amounts | \$9,947 | | | |

Calculator Solution

Deposit at the beginning of the period:

Set the calculator in the BEGIN mode

Inputs 19 8 -3,000












Solution 134,286

Deposit at the end of the period:

Set the calculator back to the END mode

Inputs 19 8 -3,000

Solution 124,339

Difference in Amounts = 134,286 – 124,339 = 9,947

12) Kenneth made a \$20,000 investment in year 1, received a \$5,000 return in year 2, made \$8,000 cash payment in year 3, and received his \$20,000 back in year 4. If his required rate of return is 8 percent, what was the net present value of his investment?

Excel Solution

| | A | B | C | D |
|----|-------------------|------------|------------------|---|
| 7 | Inputs | | | |
| 8 | Cash Flow Year 1 | (\$20,000) | | |
| 9 | Cash Flow Year 2 | \$5,000 | | |
| 10 | Cash Flow Year 3 | (\$8,000) | | |
| 11 | Cash Flow Year 4 | \$20,000 | | |
| 12 | Discount Rate | 8% | | |
| 13 | | | | |
| 14 | Solution | | | |
| 15 | Net Present Value | (\$5,882) | =NPV(B12,B8:B11) | |

13) John had \$50,000 in salary this year. If this salary is growing 4 percent annually and inflation is projected to rise 3 percent per year, calculate the amount of return he will receive in nominal and real dollars in the fifth year.

Excel Solution

| | A | B | C | D | E | F | G |
|----|--|-----------------------------|----------|----------|----------|----------|----------|
| 7 | Inputs | | | | | | |
| 8 | Present Value of Salary | \$50,000 | | | | | |
| 9 | Growth Rate | 4% | | | | | |
| 10 | Inflation Rate | 3% | | | | | |
| 11 | Number of Years | 5 | | | | | |
| 12 | | | | | | | |
| 13 | Solution | | | | | | |
| 14 | 1) Calculate Real Rate of Return | | | | | | |
| 15 | Real Return | 1% | | | | | |
| 16 | | | | | | | |
| 17 | 2) Calculate the amount of reaturn in nominal and real dollars | | | | | | |
| 18 | | | | | | | |
| 19 | Year | 0 | 1 | 2 | 3 | 4 | 5 |
| 20 | Nominal Dollars | 50,000 | 52,000 | 54,080 | 56,243 | 58,493 | 60,833 |
| 21 | Real Dollars | 50,000 | 50,485 | 50,976 | 51,470 | 51,970 | 52,475 |
| 22 | | | | | | | |
| 23 | Formula in cell G20 | =FV(\$B\$9,G19,0,-\$B\$20) | | | | | |
| 24 | Formula in cell G21 | =FV(\$B\$15,G19,0,-\$B\$21) | | | | | |

14) Becky made a \$30,000 investment in year 1, received a \$10,000 return in year 2, \$8,000 in year 3, \$11,000 in year 4, and \$9,000 in year 5. What was her internal rate of return over the five-year period?

Excel Solution

| | A | B | C | D |
|----|-------------------------|------------|--------------|---|
| 7 | Inputs | | | |
| 8 | Cash Flow Year 1 | (\$30,000) | | |
| 9 | Cash Flow Year 2 | \$10,000 | | |
| 10 | Cash Flow Year 3 | \$8,000 | | |
| 11 | Cash Flow Year 4 | \$11,000 | | |
| 12 | Cash Flow Year 5 | \$9,000 | | |
| 13 | | | | |
| 14 | Solution | | | |
| 15 | Internal Rate of Return | 10% | =IRR(B8:B12) | |

ANSWERS TO CASE APPLICATION QUESTIONS

Chapter 2 The Time Value of Money

- 1) 15 N, 7 I/Y, 3000 CHS PMT, press FV = \$75,387.06
- 2) Compounding is interest on interest in addition to interest on principal. Without compounding the loss would be 15×3210 or \$48,150. The difference between \$75,387.06 and \$48,150.00 is \$27,237.06 representing the contribution due to compounding.
- 3) 20,000 CHS PV, 70,000 FV, 20 N press I/Y = 6.46%. The rate is lower than the appropriate market rate of 7% and should be rejected.
- 4) 100,000 CHS PV (at age 65), 8,000 PMT, 17 N Press I/Y = 3.65%. This rate of return is not attractive since it is below the market rate of return and therefore the investment should be rejected.
- 5) Richard and Monica, it is apparent that you are not that familiar with time value of money and compounding concepts. Available cash has worth. It is the amount that you could receive by investing in financial assets in the marketplace. It is important that you be able to calculate this return, particularly on a compound basis. Compounding indicates interest on interest. It is a “stealth” figure which when calculated clears up any misconceptions about what is a good return. Time value of money principles and the power of compounding have indicated that the twenty-year investment offered and the annuity both have below-market returns. This wouldn't have been apparent without the calculation.

Chapter 2

The Time Value of Money

Chapter Outline

- **Basic Principles**
 - Time Value of Money
 - Compounding
 - Using a Financial Calculator
 - Present Value
 - Future Value
- **Sensitivity to Key Variables**
 - The Rule of 72
 - Compounding Periods
 - Discount Rate
 - Periods

Chapter Outline

- **Annuities**
 - Future Value of an Annuity
 - Present Value of an Annuity
 - Rate of Return on an Annuity
 - Periodic Payment of an Annuity
 - Perpetual Annuity
- **Irregular Cash Flows**
- **Inflation-Adjusted Earnings Rates**
- **Internal Rate of Return**
- **Annual Percentage Rate**
- **Chapter Summary**

Chapter Goals

- Develop a working understanding of compounding.
- Apply time value of money principles in day-to-day situations.
- Calculate values for given rates of return and compounding periods.
- Compute returns on investments for a wide variety of circumstances.
- Recognize the effect of inflation on the purchasing power of the dollar.

The Time Value of Money

- *Time value of money*: the compensation provided for investing money for a given period.
- For example:
 - You are offered the choice of \$1,000 dollars today or \$1,000 dollars two years from now. Which do you choose?
 - You would choose to receive the money today.
 - After all, if you receive the money today you can invest the money and in two years could have much more than the original \$1,000.

Compounding

- *Compounding*: the mechanism that allows the amount invested, called the principal, to grow more quickly over time.
- It results in a greater sum than just the interest multiplied by the principal.
- Once we compound for more than one period we not only receive interest on principal but interest on our interest.

Compounding, cont.

- For example:
 - Initial Principal \$2000
 - Interest Rate 10%
 - What is the principal at the end of years 1 and 2?

$$\text{Principal End of Year 1} = \$2000 \times 1.10 = \$2200$$

$$\text{Principal End of Year 2} = \$2000 \times 1.10 \times 1.10 = \$2420$$

Compounding, cont.

- Were it not for the compounding we would use a simple interest rate for two years as follows:

$$1 + .10 + .10 = 1.20$$

- The principal end of year 2 would then be:

$$\$2,000 \times (1.20) = \$2,400$$

- The \$20 difference between \$2,420 and \$2,400 represents the interest on interest.

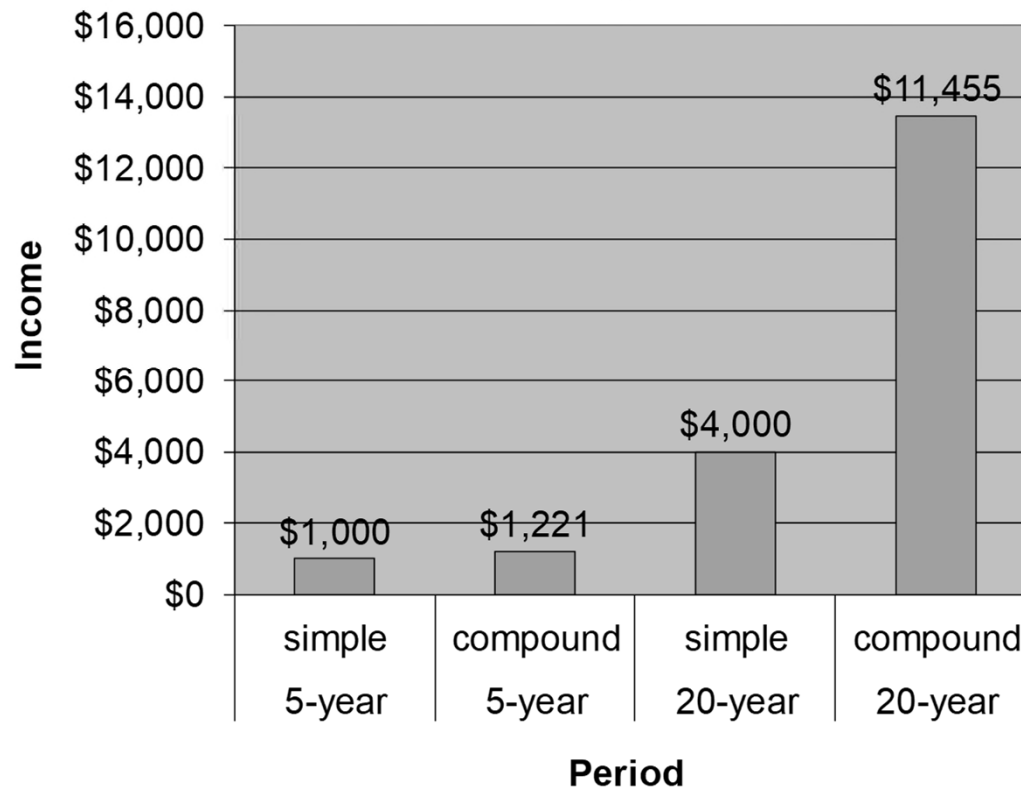
Compounding, cont.

- This table illustrates the impact of compounding over a five year period.

| Year | Beginning Principal | Ending Principal | Simple Interest Income | Compound Interest Income | Compounding Contribution |
|--------------|---------------------|------------------|------------------------|--------------------------|--------------------------|
| 1 | \$2,000 | \$2,200 | \$200 | \$200 | \$0 |
| 2 | \$2,200 | \$2,420 | \$200 | \$220 | \$20 |
| 3 | \$2,420 | \$2,662 | \$200 | \$242 | \$42 |
| 4 | \$2,662 | \$2,928 | \$200 | \$266 | \$66 |
| 5 | \$2,928 | \$3,221 | \$200 | \$293 | \$93 |
| Total | | | \$1,000 | \$1,221 | \$221 |

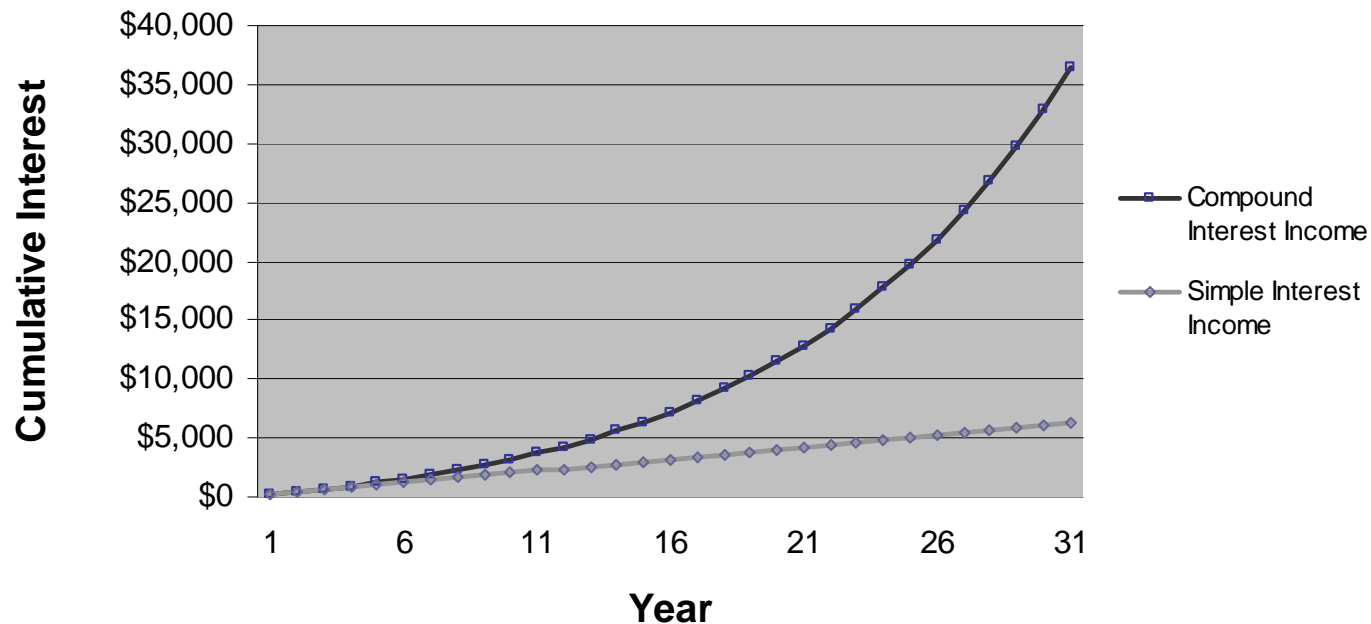
Compounding, cont.

- This figure illustrates simple versus compound interest.



Compounding, cont.

- This figure illustrates simple versus compound cumulative interest.



Using a Financial Calculator

- Time value of money and other calculations can be performed using a financial calculator.
- For example, consider the HP12C financial calculator. Five keys used in time value calculations are as follows:
 - n = The number of years or compounding periods
 - i = The rate of return or discount rate
 - PV = Present value
 - PMT = Periodic payment
 - FV = Future value

Present Value

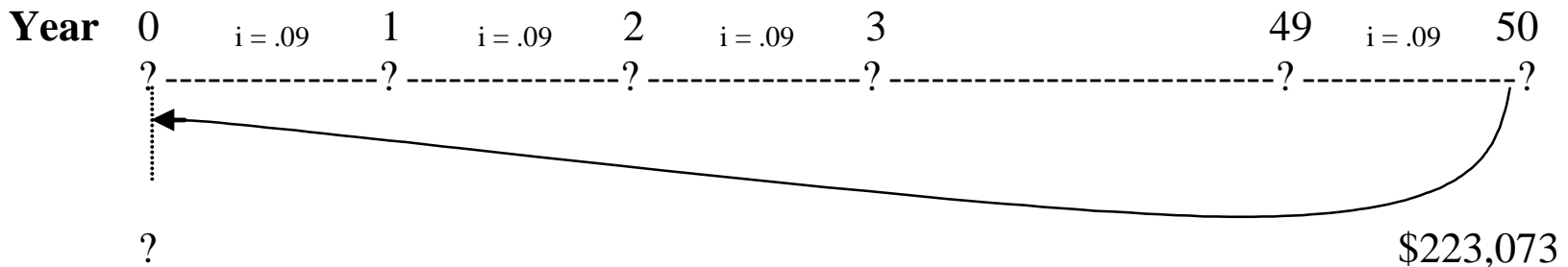
- *Present value*: The worth of a sum at the beginning of a given period of time.
- We may be offered an amount of money in the future and want to know its present value.
- We can solve for the present value using as follows:

$$PV = \frac{FV}{(1+i)^n}$$

- PV = Present Value
- FV = Future Value
- i = Interest Rate
- n = Number of Periods

Present Value, cont.

- For example:
- What is the present value of \$223,073 to be received 50 years from now if the interest rate is 9 percent?



Present Value, cont.

- Solution:

$$PV = \frac{FV}{(1+i)^n} = \frac{\$223,073}{(1+.09)^{50}} = \$3,000$$

- Calculator Solution: 50 n, 9 i, 223073 FV
- Press PV = 3000

Future Value

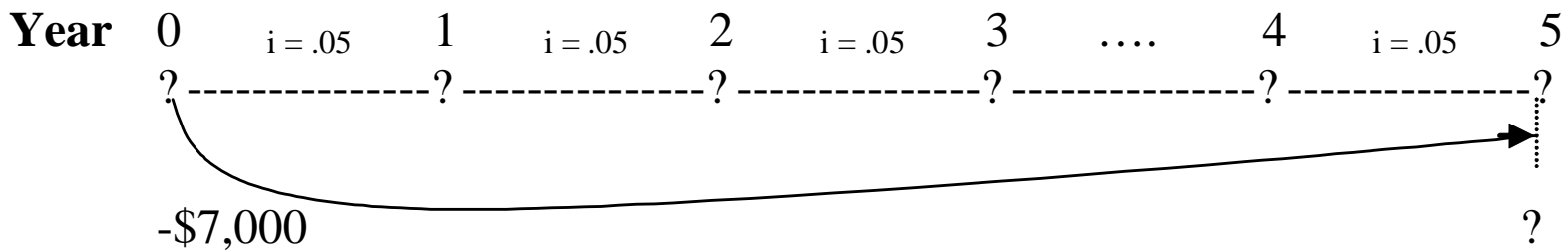
- *Future value*: The amount you will have accumulated at the end of a period.
- We can solve for the future value using as follows:

$$FV = PV(1 + i)^n$$

- PV = Present Value
- FV = Future Value
- i = Interest Rate
- n = Number of Periods

Future Value, cont.

- For example:
- If you deposit \$7,000 in a certificate of deposit for five years earning 5% annually, how much will you accumulate by the end of the period?



Future Value, cont.

- Solution:

$$FV = PV(1+i)^n = \$7,000(1+.05)^5 = \$8,934$$

- Calculator Solution: 5 n, 5 i, 7000 CHS PV
- Press FV = \$8,934

Sensitivity to Key Variables

- The interest rate and the number of time periods are the key variables for determining accumulated sums given a fixed amount deposited.
- A shift in either compounding time or in interest rate, even when relatively modest, can have a material effect on final results.

The Rule of 72

- The rule of 72 tells us how long it takes for a sum to double in value.

$$\text{Years to Double} = 72 / \text{Annual Interest Rate}$$

- For example, if the rate is 8%, then:

$$\text{Years to Double} = 72 / 8 = 9$$

Compounding Periods

- The number of compounding periods tells us how often interest on interest is calculated.
- The more often interest on interest is calculated, the greater the investment return.
- When compounding is not annual, then:
 - Divide the yearly interest rate by the number of compounding periods per year.
 - Multiply the number of years you compound by the number of compounding periods per year.

Compounding Periods, cont.

- For example, what is the future value if:
 - Initial Principal \$1,000
 - Interest Rate 8%
 - Compounding periods per year 4
- Solution:
$$FV = \$1,000 \left(1 + \frac{.08}{4} \right)^{10 \times 4} = \$2,208$$
 - Calculator Solution: 40n, 2i, 1000 CHS PV
 - Press FV = \$2,208

Discount Rate

- *Discount rate*: The rate at which we bring future values back to the present.
- Obtained by taking the rate of return offered in the market for a comparable investment.
- Sometimes designated the “present value interest factor” (PVIF).
- The higher the discount rate, the lower the present value of a future sum.
- Discount rates fluctuate for several reasons, such as inflation.

Discount Rate, cont.

- We can solve for the discount rate using the following equation:

$$(1 + i)^n = \frac{FV}{PV}$$

- PV = Present Value
- FV = Future Value
- i = Discount Rate
- n = Number of Periods

Discount Rate, cont.

- For example, what is the discount rate if:

| | |
|---------------------|----------|
| – Future Value | \$40,000 |
| – Present Value | \$20,000 |
| – Number of Periods | 9 |

- This implies: $(1 + i)^9 = \frac{FV}{PV} = \frac{\$40,000}{\$20,000} = 2$

- It follows: $i = 8\%$
- Calculator Solution: 20000 CHS PV, 40000 FV, 9n
- Press $i = 8.0\%$

Periods

- We may wish to solve for the number of periods associated with the investment.
- We can solve for the number of periods using the same method we used to solve for the discount rate.

Periods, cont.

- For example, how many periods are there if:

| | |
|-----------------|----------|
| – Future Value | \$19,672 |
| – Present Value | \$10,000 |
| – Discount Rate | 7% |

- This implies: $(1 + .07)^n = \frac{FV}{PV} = \frac{\$19,672}{\$10,000} = 1.9672$

- It follows: $n = 10$ years
- Calculator Solution: 7i, 10000 CHS PV, 19672 FV
- Press $n = 10$ years

Annuities

- *Annuity*: a series of payments that are made or received.
- *Ordinary annuity*: When annuity payments are made at the end of the period.
- *Annuity due*: When payments are made at the beginning of the period .
- We can calculate the future and present value of annuities through using formulas that accommodate multiple cash flows.

Future Value of an Annuity

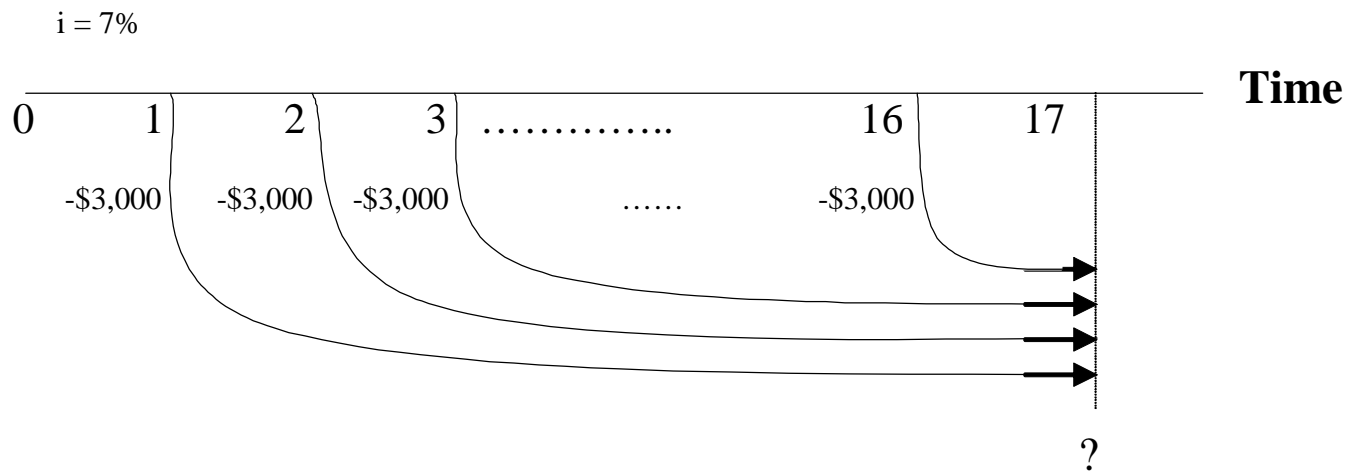
- We can solve for the future value of an annuity using the following equation:

$$FVA = PMT \times \frac{(1 + i)^n - 1}{i}$$

- FVA = Future Value of an Annuity
- PMT = Annual Payment
- i = Interest Rate
- Number of Periods

Future Value of an Annuity, cont.

- For example:
- If the annuity payments are \$3,000 at the end of each year and the interest rate is 7%, what is the future value of the annuity in 17 years?



Future Value of an Annuity, cont.

- Solution:

$$FVA = PMT \times \frac{(1+i)^n - 1}{i} = \$3,000 \times \frac{(1+.07)^{17} - 1}{.07} = \$92,521$$

- Calculator Solution: 17n, 7i, 3000 CHS PMT
- Press FV = 92521

Present Value of an Annuity

- We can solve for the future value of an annuity using the following equation:

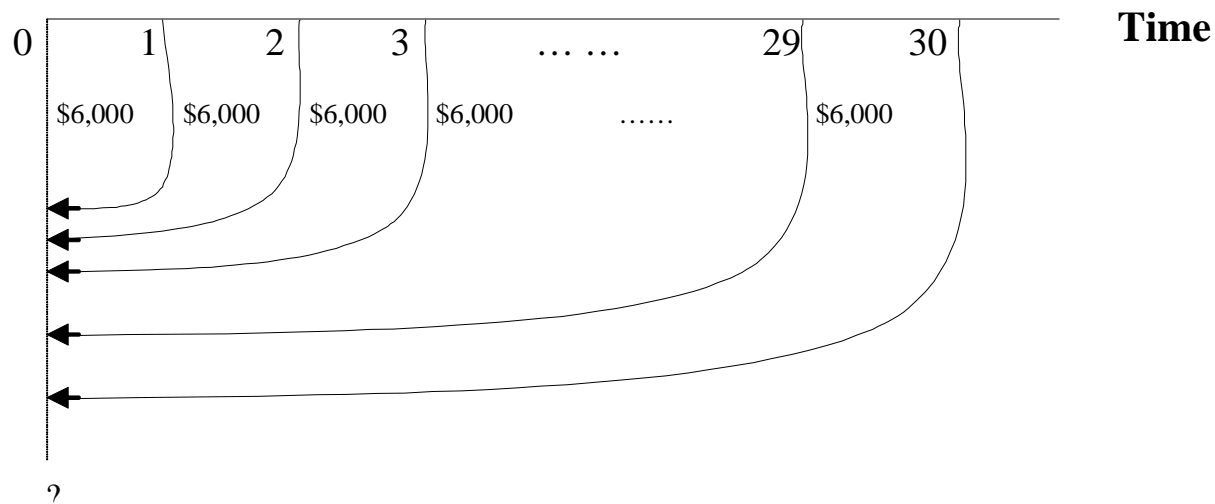
$$PVA = \sum_{t=1}^n \frac{PMT}{(1+i)^t}$$

$$PVAD = PVA \times (1+i)$$

- PVA = Present Value of an Annuity
- PVAD = Present Value of an Annuity Due
- PMT = Annual Payment
- i = Interest Rate
- Number of Periods

Present Value of an Annuity, cont.

- For example:
- If the annuity payments are \$6,000 at the beginning of each year for 30 years and the discount rate is 7%, what is the present value?



Present Value of an Annuity, cont.

- Solution:

$$PVAD = \sum_{t=1}^n \frac{PMT}{(1+i)^t} \times (1+i) = \sum_{t=1}^{30} \frac{\$6,000}{(1+.07)^t} \times (1+.07) = \$79,666$$

- Calculator Solution: 30n, 7i, 6000 PMT, g BEG
- Press PV = 79666

Rate of Return on an Annuity

- If we know the cash flows associated with an annuity we can solve for the discount rate.
- For example, if the PVA is \$100,000 and the annuity payments are \$8,000 for an ordinary 20-year annuity, what is the discount rate?
- Solution: $\$100,000 = \sum_{t=1}^{20} \frac{\$8,000}{(1+i)^t}$. It follows $i = 5\%$.
- Calculator Solution: 20n, 100000 CHS PV, 8000 PMT
- Press $i = 5\%$

Periodic Payment of an Annuity

- We can solve for annuity payment.
- For example, if the PVA is \$25,000, the discount rate is 8%, what are the annuity payments associated with an ordinary 8-year annuity?

- Solution: $\$25,000 = \sum_{t=1}^8 \frac{PMT}{(1+.08)^t}$. It follows $PMT = \$4,350$.

- Calculator Solution: 8n, 8i, 25000 PV
- Press PMT = 4350

Perpetual Annuity

- *Perpetual annuity*: a stream of payments that is assumed to go on forever.
- The present value of a perpetual annuity is calculated as

$$PVA_P = \frac{PMT}{i}$$

- For example, if the perpetual annuity is equal to \$5 and the interest rate is 9%, then the value is as follows:

$$PVA_P = \frac{PMT}{i} = \frac{\$5}{0.09} = \$55.56$$

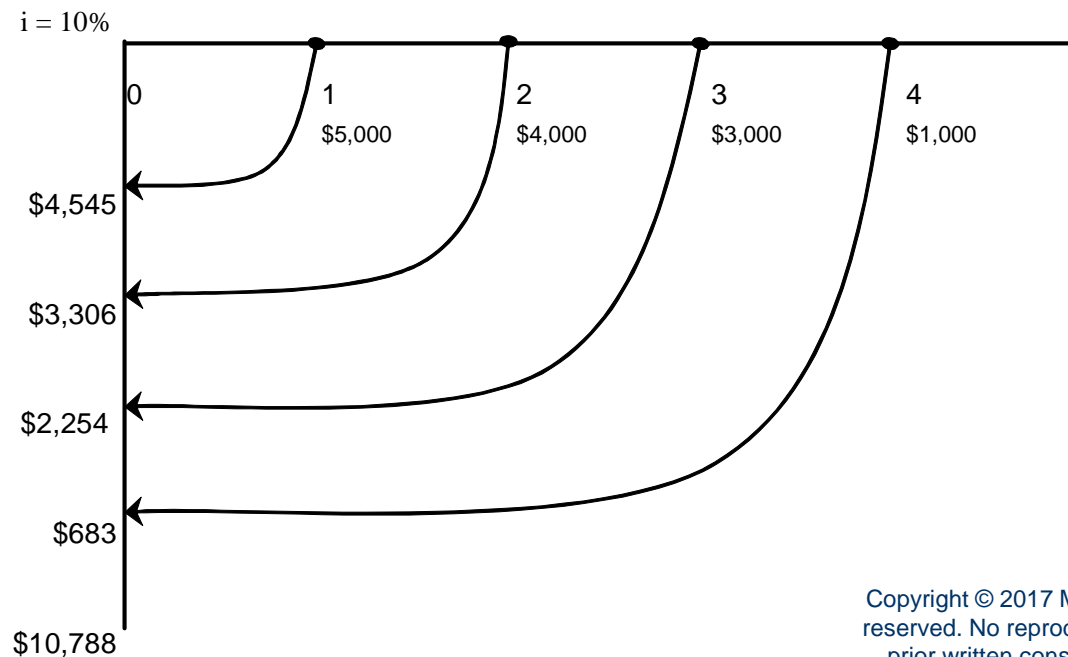
Irregular Cash Flows

- In many instances cash flows differ across periods. We can call these differing payments *irregular cash flows*.
- To calculate the present and future values, each cash flow considered separately.
- For example:
- An investment supplies \$5,000 in year 1, \$4,000 in year 2, \$3,000 in year 3 and \$1,000 in year 4. What is the value of the investment if the interest rate is 10%?

Irregular Cash Flows, cont.

- Calculator Solution:

- 0CHS gCF0, 5000 gCFj, 4000 gCFj, 3000 gCFj, 1000 gCFj, 10i
- Press f NPV = \$10,788



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Inflation-Adjusted Earnings Rates

- *Inflation*: the rate of increase in prices in our economy or in specific items.
- Inflation can distort earnings results.
- *Real Return*: the inflation-adjusted return.
- *Nominal Return*: the return without inflation adjustment.
- A decline in purchasing power occurs when *real dollars* decrease.

Inflation-Adjusted Earnings Rates, cont.

- We can calculate the real return as follows:

$$RR = \left(\frac{1+r}{1+i} - 1 \right) \times 100$$

- RR = Real Return
- R = Investment Return
- i = Inflation Rate

Inflation-Adjusted Earnings Rates, cont.

- For example:
 - The current value of an individual's savings is \$500,000.
 - The \$500,000 provides \$35,000 this year, which is growing 3 percent annually.
 - Inflation is projected to rise 5 percent per year.
 - What is the value of the nominal and real dollars provided today and each of the next five years?

Inflation-Adjusted Earnings Rates, cont.

- Solution, year 1:

$$\text{Nominal Return Year 1} = 35,000 \times 1.03 = 36,050$$

$$\text{Real Return Year 1} = \$35,000 \times \left(\frac{1 + 0.03}{1 + 0.05} \right) = 34,335$$

- Solution, all years:

| Years | 0 | 1 | 2 | 3 | 4 | 5 |
|-----------------|--------|--------|--------|--------|--------|--------|
| Nominal Dollars | 35,000 | 36,050 | 37,132 | 38,245 | 39,393 | 40,575 |
| Real Dollars | 35,000 | 34,335 | 33,683 | 33,043 | 32,415 | 31,791 |

Internal Rate of Return

- *Internal rate of return* (IRR): discount rate that makes the cash inflows over time equal to the cash outflows.
- It combines all cash outflows and inflows:
 - Outflows: Usually initial outlays to purchase the investment plus any subsequent losses.
 - Inflows: The income on the investment plus any proceeds on sale of the investment.

Internal Rate of Return, cont.

- Example:
 - Lena had a stock that she purchased for \$24. She received dividends 1 and 2 years later of \$0.80 and \$0.96, respectively, and then sold her investment in year 3 for \$28. What is her IRR?
- Calculator Solution:
 - 24 CHSgCF0, 0.80gCFj, 0.96gCFj, 28gCFj
 - Press fIRR = 7.7%

Annual Percentage Rate

- *Annual percentage rate (APR)*: an adjusted interest on a loan.
- The federal Truth in Lending Act mandates that this rate be disclosed on all loans so that consumers can compare the rates offered by different lenders.
- The APR incorporates many costs other than interest that make its rate different from the one included in a lending contract.
- Costs include loan processing fees, mortgage insurance, and points.

Chapter Summary

- The time value of money enables you to make correct decisions when current or future amounts need to be established or when deciding which alternative is best. It allows impartial comparison of past or future performance or values.
- Cumulative sums are highly sensitive to the number of compounding periods and to the rate of return used.
- It is essential when making decisions to know the present value, the future value, the discount rate for lump sums and for annuities.

Chapter Summary, cont.

- Real rates of return are those adjusted for inflation.
- The internal rate of return (IRR) is the one most commonly used to compare the return on investments that have differing inflows and outflows over time.